FRACTURE MODEL FOR QUASIBRITTLE MEDIA

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The behavior of materials subjected to different loads depends substantially on the nature of the plastic flow that is determined by the internal structure of the material, the presence of different defects therein, the inhomogeneities, etc. For instance, the mechanism of plastic flow of metals is associated mainly with the motion of dislocations [1, 2]. The fracture of brittle media is governed by the presence of microcracks therein, resulting in stress concentration at the crack apex, where a plastic zone is formed [3]. Consequently, the plastic flow in brittle media turns out to be microscopically inhomogeneous, which distinguishes them substantially from plastic media. The need to take account of the presence of internal inhomogeneities in describing the deformation and fracture of brittle media is mentioned in [4]. It is assumed in this paper that microcracks are the carriers of such inhomogeneities in brittle media.

A model of the plastic flow of brittle media prior to fracture was first proposed in [5], where the mutual displacement of edges of the crack originating because of the presence of a plastic zone near the tip underlies the plastic flow mechanism. However, it was assumed in [5] that all cracks are of identical size. In this paper, this model is extended to the case of microcracks of different length. Also taken into account is the microcrack orientation relative to the principal stress axes. An equation is obtained that describes material behavior prior to fracture for different kinds of loads. A fracture criterion is proposed that is based on the representation of fracture as the intersection of growing cracks. Within the framework of the proposed approach, in particular the dependence of the mean size of pieces of the fractured rock on the loading parameters, is determined successfully. The results of solving the proposed equations are compared with experimental results on the slow compression of colophony specimens [4, 6].

1. Let us consider the macroscopic deformations of a cracked medium. These deformations can be represented as the sum of elastic and plastic deformations associated with the presence of a plastic zone near the crack apex. The elastic deformations are distorted because of the presence of cracks, resulting, in particular, in a dependence of the elastic moduli on the fracturing [7]. However, these effects are not considered in this paper. The plastic flow is associated with crack growth resulting in motion of the plastic zone through the bulk of the specimen.

The magnitude of the plastic deformations can be calculated analogously to the derivation of the Orovan derivation within the framework of the dislocation model [1, 2]. To do this, we examine a specimen section with linear dimension L. The shear deformations of this specimen, occurring during motion of one crack, will be determined by the relationship $\gamma = (\Delta/L)\cdot(s/L)$, where Δ is the displacement of the crack edges related to the plastic flow at its apex, and s is the change in the length of this crack. Let us note that the quantity Δ/L determines the angle of shear from one crack, while s/L is the relative size of the zone included in the deformations. Differentiating the relationship obtained with respect to time, and summing over all cracks, we obtain a relationship for the shear deformation rate

$$\frac{d\gamma}{dt} = \frac{1}{G}\frac{d\tau}{dt} + \cos 2\varphi\cdot\Delta\int_0^\infty v(l)\, n(l, t)\, dl, \tag{1.1}$$

where γ is the macroscopic shear deformation; τ is the tangential stress; Δ is the crack edge displacement; and v is the rate of crack growth. The quantity $n(\ell, t)d\ell$ defines the number of crack boundaries that intersect a unit surface whose linear dimensions are in the interval between ℓ and $\ell + d\ell$. In deriving this relationship it was assumed that the value of Δ is identical for all growing cracks. It is reasonable to select the magnitude of the critical

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opening of a crack in the Leonov–Panasyuk model [3, 8] as an estimate of the value of Δ. The factor cos 2φ takes account of the different orientation of the principal elastic and inelastic deformation axes, where φ is the difference between the slip and Coulomb angles. We shall henceforth omit this factor.

The function n(ℓ),t) is normalized relative to the total number of cracks intersecting the unit surface. Assuming the number of cracks remains constant, we obtain an equation for the function n(ℓ), t)

$$\frac{\partial n(l, t)}{\partial t} + \frac{\partial}{\partial l}\{n(l, t)\,v(l)\} = 0, \tag{1.2}$$

which is essentially a continuity equation for the crack size distribution density.

The crack growth rate v(ℓ) depends on both the crack length and the magnitude of the external loads. We shall assume that under compression the crack growth rate is determined by the effective tangential stress [9, 10] that takes account of crack edge interaction:

$$\tau_{\mathrm{eff}} = |\sigma_\tau| = \mu|\sigma_n|,\ |\sigma_\tau| = -\frac{\sigma_1-\sigma_3}{2}|\sin 2\theta|,\ \ |\sigma_n| = -\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta, \tag{1.3}$$

where θ is the angle between the direction σ_1 and the crack plate; σ_τ and σ_n are the tangential and normal stresses in the crack; and μ is the friction coefficient between the crack edges. In this paper the simplest dependence between the crack growth rate and ℓ and τ_{eff} will be used:

$$v(l, \tau_{\mathrm{eff}}) = \begin{cases} 0, & K < K_0, \\ v_p, & K \geqslant K_0, \end{cases} \tag{1.4}$$

where $K = \tau_{\mathrm{eff}}\sqrt{\ell}$ is the stress concentration factor at the crack apex. According to the dependence (1.4), only cracks whose length exceeds the quantity $K_0^2/(\tau_{\mathrm{eff}}^2)$ can grow. The growth rate of all such cracks is identical here and equal to v_p.

It follows from (1.3) and (1.4) that the cracks of maximal length grow most intensively in directions close to the directions of the maximal effective stresses governed by the condition cot 2θ = μ. For simplicity we shall temporarily neglect the spread in the initial directions of the microcracks that start to grow first. Such a spread will be taken into account below, which will result in a change in the function of the number of growing cracks.

The relationships (1.1)-(1.4) govern the dynamics of shear deformation up to fracture time. We take the condition of equal distances between growing cracks and their mean length $\bar{\ell}_m$ as fracture criterion

$$l_m N_m^{-1/2} = 1, \tag{1.5}$$

where N_m is the number of growing cracks determined by the expression

$$N_m = \int_{K_0^2/\tau_{\mathrm{eff}}^2}^{\infty} n(l, t)\,dl. \tag{1.6}$$

Then the mean length of the growing crack will be given by the relationship

$$\bar{l}_m = N_m^{-1}\int_0^{\infty} l n(l, t)\,dl. \tag{1.7}$$

Taking (1.6) into account, we rewrite (1.1) in the form

$$\frac{d\gamma}{dt} = \frac{1}{G}\frac{d\tau}{dt} + \Delta v_p N_m. \tag{1.8}$$

A fracture criterion of the form (1.5) is correct in the case when the crack growth is accompanied by their effective intersection. Such a situation should be observed, in particular, in fracture due to shear, when the most intensive crack growth occurs in two mutually intersecting directions. Use of the criterion (1.5) permits not only establishment of the fracture time but also determination of the mean size of the fractured rock pieces.

2. Before solving the equations describing deformation of the medium for a specific loading case, they must be reduced to dimensionless form, which permits investigation of their fundamental properties. For this, we introduce the function

$$N(l, t) = \int_l^\infty n(l', t)\, dl',$$

governing the concentration of cracks whose size exceeds ℓ. It follows from (1.2) that the total number of cracks $N_t = N(0, t)$ remains constant during deformation and is a characteristic of the medium. It follows from (1.1) that the characteristic time governing the plastic deformation rate is the quantity $t_* = (N_t \Delta v_p)^{-1}$. We assume that the condition $N_t^{-\frac{1}{2}} \ell_0 \ll 1$ is satisfied in the initial state, i.e., the medium is far from the fracture point. Then we select the characteristic length $\ell_* = v_p t_* = (N_t \Delta)^{-1}$. Finally, it follows from (1.4) that the characteristic strength parameters $\tau_0 = K_0/\sqrt{\ell_0}$ must be selected, where ℓ_0 is the characteristic dimension of the microcracks in the initial state.

Introducing the dimensionless quantities

$$T = t/t_*,\ L = l/l_*,\ s = \tau_{eff}/\tau_0,\ f_m(s) = N_m/N_t,\ g = \tau_0/G,\ D = N_t^{1/2}\Delta, \tag{2.1}$$

we can represent (1.8) and the fracture condition (1.5) in the dimensionless form

$$d\gamma/dt = g ds/dt + f_m(s); \tag{2.2}$$

$$f_m(s)\,\overline{L}_m^2 = D^2. \tag{2.3}$$

It is seen from the relationships (2.1)-(2.3) that the nature of the deformation prior to fracture is determined by the single dimensionless parameter g that governs the magnitude of the elastic deformations. Moreover, it is necessary to give the deformation law [the dependence $\gamma(T)$ or $s(T)$, e.g.]. At the same time, the fracture time itself will be determined by the dimensionless parameter D, the ratio between the crack edge displacement and the spacing between the cracks.

To solve the equations describing deformation of the medium, the nature of the external loading must be given. Let us consider the case of a constant deformation rate $\dot{\gamma}$ = const. The single dimensionless parameter characterizing the external loading will be

$$R = \dot{\gamma}/(N_t \Delta v_p) \equiv d\gamma/dT.$$

Then we write (2.2) as

$$g ds/dT = R - f_m(s).$$

The solution of this equation has the form

$$T = g \int_0^s \frac{ds'}{R - f_m(s')} \tag{2.4}$$

and yields the dependence $s(T)$ in implicit form.

As seen from the definition (2.2), the function $f_m(s) \ll 1$. Then it follows from (2.4) that two cases must be examined: $R > 1$ and $R < 1$. In the former case the denominator in the integrand in (2.4) never vanishes. For small stresses $f_m(s) \ll 1$ the dependence $s(T)$ is linear in nature:

$$s(T) = RT/g. \tag{2.5}$$

As the stress grows, $f_m(s)$ grows and the plastic flow starts to have a noticeable effect. A nonlinear section hence appears on the curve $s(T)$. For a further growth of s, $f_m(s)$ reaches its limit value and the dependence $s(T)$ again becomes linear: $s(T) = (R - 1)T/g$. In this case the plastic flow results in a diminution in the modulus G [$G_{eff} = G(1 - 1/R)$] and a dependence of the effective elastic modulus G_{eff} on the deformation rate appears.

In the second case ($R < 1$) the role of the plastic flow turns out to be more substantial. For small s the dependence $s(T)$ is elastic in nature, given by (2.5). As the stress grows

$f_m(s)$ grows and the denominator in (2.4) tends to zero. Here s approaches asymptotically to its limit value s_{max} defined by the implicit expression

$$f_m(s_{max}) = R. \tag{2.6}$$

In order to obtain a specific form of the deformation curves, the fracture time and the mean size of the pieces of fractured rock, it is necessary to give the specific form of the crack size distribution. We take the uniform distribution

$$n_0(l) = (N_t/l_0)\eta(l_0 - l) \tag{2.7}$$

as such a distribution, but it describes the crack distribution in a broad interval ℓ poorly. However, in the case of small deformation rates (R << 1) the contribution to the deformation is given by a small fraction of the cracks near the maximal dimension ℓ_0. Consequently, in this case a distribution of the form (2.7) turns out to be sufficiently general.

In the case of the initial distribution (2.7), it is easy to obtain the expression

$$f_m(s) = (1 - s^{-2})\eta(1 - s^{-2}) \tag{2.8}$$

for the quantity f_m defined by (2.1). It is taken into account here that cracks are not established for a constant deformation rate; consequently, all the cracks are growing, where the initial dimension exceeds the quantity K_0^2/τ_{eff}^2. The fraction of these cracks out of their total number evidently equals $(1/\ell_0)(1 - K_0^2/\tau_{eff}^2)$, from which we have (2.8).

Substituting (2.8) into (2.4), and integrating, we obtain

$$T = \begin{cases} g\dfrac{1}{R}s, & s \leqslant 1, \\ g\left\{\dfrac{1}{R} + 1 - s + \dfrac{1}{2\sqrt{1-R}}\ln\left[\dfrac{1+s\sqrt{1-R}}{1-s\sqrt{1-R}}\dfrac{1-\sqrt{1-R}}{1+\sqrt{1-R}}\right]\right\} & s \geqslant 1. \end{cases}$$

The maximal stress governed by the relationship (2.6) is $s_{max} = (1 - R)^{-\frac{1}{2}}$. For small deformation rates (R << 1) the excess of the maximal value s_{max} over the elastic limit s = 1 turns out to be quite small: $s_{max} - 1 = R/2$. It is hence seen that experimental observation of the "transition" zone where the transition from the elastic to the plastic flow regime occurs is quite difficult.

Let us turn to determination of the magnitude of the fracture rock pieces. Fracture can occur on the almost elastic section, in the transition zone, and on the asymptotic. The case of fracture on the linear section corresponds to the condition

$$f_m(s_p) \ll R, \tag{2.9}$$

where s_p is the stress at the time of fracture, and the fracture condition takes the form

$$s_p - 1 = (2R^2D^2/g^2)^{1/3}. \tag{2.10}$$

Substituting (2.8) and (2.10) into (2.9), we see that fracture occurs on the linear section if the condition $R \gg 16D^2/g^2$ is satisfied. In this case, we obtain the following expression for the mean size of the pieces: $\bar{L}_m = (gD^2/4R)^{1/3}$.

The second case that allows analytic examination corresponds to fracture on the asymptotic. The condition $s_{max} - s_p \ll s_{max} - 1 \approx R/2$ should be satisfied here. Consequently, the mean dimension of the pieces of fractured rock is found simply from the condition $f_m(s_{max}) = R$. Then

$$\bar{L}_m = D/\sqrt{R}. \tag{2.11}$$

The condition for applicability of this approximation will be determined by the inequality $R \ll 4D^2/g^2$.

Therefore, for small deformation rates, the mean size of the pieces of fractured rock turn out to be inversely proportional to the square root of the deformation rate.

3. Let us examine the influence of microcrack orientation. We assume that there is no isolated direction in the medium prior to loading. In this case the microcrack angle

distribution should be isotropic: $n_0(\ell, \theta) = (1/\pi)n_0(\ell)$ (θ is the angle between the plane of the crack and the principal direction σ_1).

At each instant the angular density of the growing cracks in the direction θ is

$$N_m(\theta) = \frac{N_t}{\pi}\left(1 - \frac{\tau_0^2}{\tau_{eff}^2(\theta)}\right)\eta\left(1 - \frac{\tau_0^2}{\tau_{eff}^2(\theta)}\right), \tag{3.1}$$

where $\tau_{eff}(\theta)$ is taken from (1.3).

Integrating (3.1) with respect to the angles θ, we obtain the total number of growing cracks. If the condition $R \ll 1$ is satisfied, then a narrow domain of angles $|\theta - \theta_0| \ll 1$ operates near the directions of the maximal effective stresses governed by the relationship $\cot 2\theta_0 = \mu$. In this case, the total number of growing cracks is given by the formula

$$N_m(\tau_{eff}) = \frac{8}{3\pi\sqrt{2}} N_t \left(\frac{\tau_{eff} - \tau_0}{\tau_0}\right)^{3/2}\left[\frac{\tau_0\sqrt{1+\mu^2}}{\frac{|\sigma_1 - \sigma_3|}{2}}\right]^{1/2}, \tag{3.2}$$

where $\tau_{eff} = \tau_{eff}(\theta_0)$ is the effective stress in the direction θ_0. In the dimensionless notation of Sec. 2, we have

$$f_m(s) = \alpha(s-1)^{3/2}\left[\frac{\tau_0\sqrt{1+\mu^2}}{\frac{|\sigma_1 - \sigma_3|}{2}}\right]^{1/2}, \tag{3.3}$$

where $\alpha = 8/(3\pi\sqrt{2})$ is a numerical coefficient.

Let us note that without taking account of the orientation for $s - 1 \ll 1$, $f_m(s)$ would be a linear function of $s - 1$, as seen from (2.8). The nonlinearity of $(s - 1)$ in (3.3) is due to the dependence of the width of the angle within which crack growth is possible, on the stress. Moreover, an additional effect occurs because of the presence of the last factor in (3.3), in whose denominator is $(|\sigma_1 - \sigma_3|)/2 \approx \tau_0 + \mu P$. For strong hydrostatic reduction ($P \gg \tau_0$) this factor diminishes the total number of growing cracks substantially. If the reduction is negligible, then this factor is close to one. On the whole, the influence of microcrack orientation reduces to a change in the dependence of the number of growing cracks on the stress.

For a dimensionless greatest possible stress we obtain $s_{max} - 1 \approx (R/\alpha)^{2/3}$ from (2.4) with (3.3) taken into account. Here, if the condition

$$\frac{s_p - 1}{s_{max} - 1} \approx \left[\left(\frac{5}{2}\frac{D}{g}\right)^6 \frac{\alpha^4}{R}\right]^{2/21} \ll 1 \tag{3.4}$$

is satisfied, the fracture will occur on the linear section of the deformation curve. In this case the mean size of a piece of fracture rock is

$$d/l^* \approx \left(\frac{144\pi^2}{125}\right)^{1/7}\left(\frac{g^3 D^4}{R^3}\right)^{1/7}\left(\frac{|\sigma_1 - \sigma_3|}{2\tau_0\sqrt{1+\mu^2}}\right)^{1/7}. \tag{3.5}$$

When $1 - \left[\left(\frac{5}{2}\frac{D}{g}\right)^6 \frac{\alpha^4}{R}\right]^{\frac{2}{21}} \ll 1$, the fracture occurs on the asymptotic and the mean size of the piece is determined by (2.11).

4. We turn to a comparison of the results of the model proposed and the experimental data in [4], where slow compression of cylindrical specimens was performed at a constant axial deformation rate $\dot{\varepsilon}_1 = \text{const}$ and constant radial reduction $\sigma_2 = \text{const}$, achieved by plastic flow of the metal holder. Such geometry differs somewhat from the case considered of a constant shear deformation rate under a constant normal pressure. However, assuming linearity of the dependence of the bulk deformations on the pressure and taking account of (1.1), the following expression is easily obtained for the deformation rate:

$$\frac{3}{2}\dot{\varepsilon}_1 = \left(\frac{1}{3K} + \frac{1}{G}\right)\frac{1}{\sqrt{1+\mu^2} - \mu}\frac{d\tau}{dt} + \Delta v_p N_m, \tag{4.1}$$

where K is the bulk compression modulus.

This equation is actually equivalent to (1.1), and hence the solutions of Sec. 3 are applicable to it.

The quantity Δ can be estimated by using the connection between the maximal critical opening and the limit stress concentration factor [3]. Thus, for media of the epoxy resin type we obtain $\Delta \sim 10^{-6}$ cm. The ratio $g = \tau_0/G$ for colophony is about 10^{-3} if the shear strength is taken as τ_0; we take $\sim 10^3$ cm^{-2} as N_t [11]. The question on estimating the crack growth rate under slow deformation when the crack growth occurs because of plastic flow at the apex is more complicated. Consequently, it is reasonable to assume that an increase in the crack length occurs because of absorption of the dislocations. In this case we obtain the estimate

$$v_p \sim N_d^{1/2} b v_d \sim 10^{-4} v_d,$$

where N_d and v_d are the density and velocity of dislocation motion, and b is the Burgers vector. Taking the limit value $v_d \sim 10^5$ cm/sec for v_d, we have $v_p \sim 10$ cm/sec.

Two deformation rates $\dot{\varepsilon}_1 = 3 \cdot 10^{-6}$ and $9 \cdot 10^{-5}$ sec^{-1} were investigated in [4]. Then, by using the model parameters presented above and taking account of (4.1), we find $R = 4.5 \cdot 10^{-4}$ and $1.4 \cdot 10^{-2}$.

For these values of the parameters the left side of the inequality (3.4) equals 0.3, which affords a foundation for determining the mean size of the pieces by using (3.5). For the deformation rates $\dot{\varepsilon}_1 = 3 \cdot 10^{-6}$ and $90 \cdot 10^{-6}$ sec^{-1} used in experiment [4], (3.5) yields the mean size 53 and 11 mm, respectively, for the pieces. The experimental dimensions have a substantially lower value, 13 and 3.9 mm, respectively. Such a strong discrepancy between the results is apparently due to the additional granulation of the already fractured medium described in [6]. A condition for applicability of the formulas obtained, (3.5) and (2.11), is a drop in the stresses right after the intersection of cracks, while the loading was executed in the experiment until the attainment of a definite value of deformation and the piece size is unknown at the time of fracture. It is clear that under these conditions (2.11) for fracture on the asymptotic should yield results closer to the experimental values. The mean size of pieces computed by (2.11) equal 14 and 2.5 mm, respectively, for the two deformation velocities, which is in good agreement with the results of the experiment. In the case of fracture on the asymptotic the mean size of a piece given by (2.11) is independent of the total number of cracks N_t, which we do not know in advance.

It should be noted that experiments with a greater number of deformation rates are desirable, since it is impossible to confirm the dependences obtained by two experimental points. It can be expected that for small deformation rates the dependence of the mean size of a piece on the deformation velocity will be close to $\sim\dot{\varepsilon}^{-1/2}$, and for higher rates to $\sim\dot{\varepsilon}^{-3/7}$

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VOID FORMATION, EQUATIONS OF STATE, AND STABILITY OF SUPERPLASTIC DEFORMATION OF MATERIALS

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The term "superplastic" is taken to indicate a state and behavior of materials in which there is a significant increase in the capacity for deformation (sometimes by hundreds or thousands of a percent) without signs of macroscopic fracture and with a simultaneous decrease in the flow stress [1, 2]. It has now been established that superplasticity is seen in nearly all engineering alloys based on iron, nickel, titanium, and aluminum. This includes hard-to-deform tool and heat-resistant steels and alloys, composites, cermets, and ceramics.

Two main types of superplasticity are traditionally recognized: structural (isothermal) superplasticity, due to an ultrafine structure; superplasticity associated with a transformation in the phase-transition temperature range. The features of the manifestation of superplasticity demonstrate the need to allow for the structure of the material, while the strong effect of strain rate on superplasticity regimes indicates the need for proper description of relaxation processes. The present article studies the effect of the structure of the material and void formation on superplastic behavior and its stability.

The main structural sign of superplastic deformation for a given temperature-rate regime is mass displacement of grains of the "overflow" type. The massive nature of such displacements ensures an exceptionally high degree of plasticity without appreciable deformation of individual grains. The development of flow which is almost "hydrodynamic" in character with respect to each specific grain is naturally connected with the appearance of free volume. It is known that plastic deformation is accompanied by the formation of microcracks and voids. This phenomenon has been given the name "plastic loosening" [3]. In [4, 5] a study was made of the mechanism of superplasticity accompanied by intensive void formation. It was shown that the presence of voids and microcracks is an important structural factor which ensures an unusually high degree of plastic strain.

As a parameter determining the volume concentration and primary orientation of voids and microcracks, we might use the symmetrical tensor $p_{ik} = n\langle s_{ik}\rangle$, where n is the number of microcracks in a unit volume, while the "microscopic" quantity

$$s_{ik} = s\nu_i\nu_k \qquad (1)$$

characterizes the volume and orientation of a normal-rupture microcrack with the base $S_D = S_D\nu$ and the vector $b = b\nu$ for the jump in displacements [6]. The volume of the microcrack is $s = \mathrm{Sp}\, s_{ik} = S_D b$, while the structure of the tensor s_{ik}, which is bilinear in relation to the components of the unit vector ν, is similar, for example, to the structure of the orientation tensor in the physics of polymers and pure liquids [7].

The laws of crack formation in polycrystalline solids are related to the considerable heterogeneity of the microstructure [8]. Dislocation pileups, boundaries of blocks, and intergranular boundaries are nuclei of microcracks in metals. Nuclei exceeding a certain critical size are capable under certain conditions of increasing their volume and develop-

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